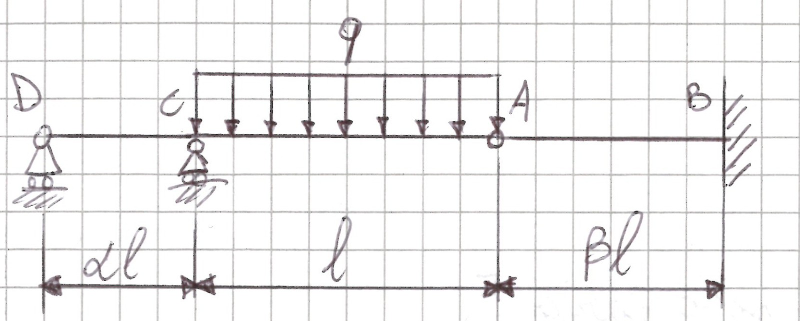
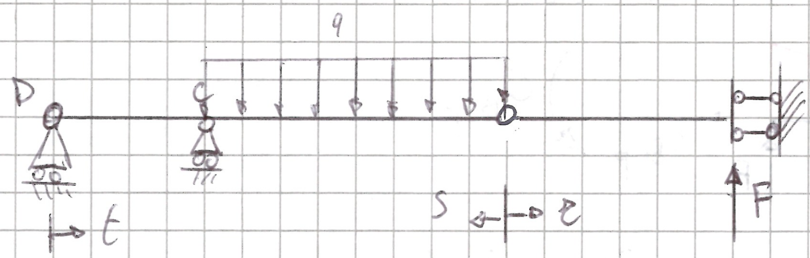
Esercizio 2.02

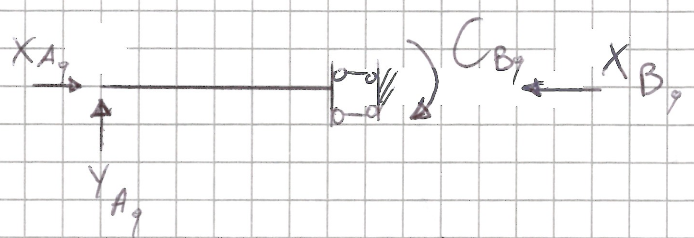




Calcolo la reazione iperstatica F con il teorema di Castigliano.

Considero il solo carico q.

$\overline{AB}$



$\uparrow^+]\ Y_{A_q} = 0$

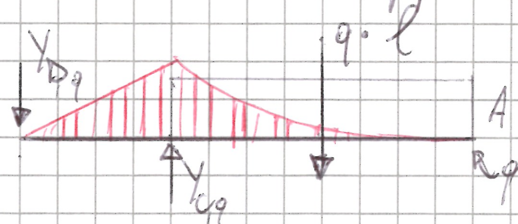
$\rightarrow^+]\ X_{A,q} = X_{B,q} \rightarrow X_{B_q} = 0$

$\circlearrowleft^+_B]\ C_{B_q} = -Y_{A_q} \cdot \beta l = 0$

Se osservo $\overline{DCA}$, $X_A = 0$

Il tratto $\overline{AB}$ è scarico se applico il solo carico q.

$\overline{DCA}$



qui è tutto scarico da come abbiamo calcolato precedentemente.

$\uparrow^+]\ Y_{Dq} + ql = Y_{Cq}$

$(\rightarrow^+]\ X_{Aq} = 0)$

$$\circlearrowleft^+_C]\ Y_{D,q} = \frac{ql \cdot \frac{\cancel{l}}{2}}{\alpha \cancel{l}} = \frac{ql}{2\alpha} \Rightarrow Y_{C_q} = \frac{2\alpha ql + ql}{2\alpha} = ql\frac{(2\alpha+1)}{2\alpha}$$

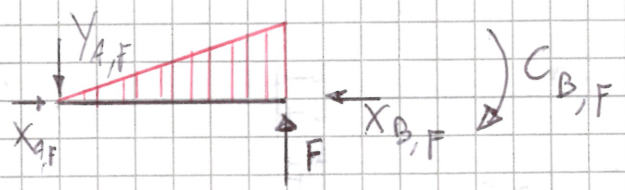
$$M_{f_{q,AB}} = \varnothing$$

$$M_{f_{q,AC}} = -\frac{1}{2} q\, s^2$$

$$M_{f_{q,DC}} = -\frac{ql}{2\alpha} \cdot t$$

- Considero ora il solo carico F.

$\overline{AB}$



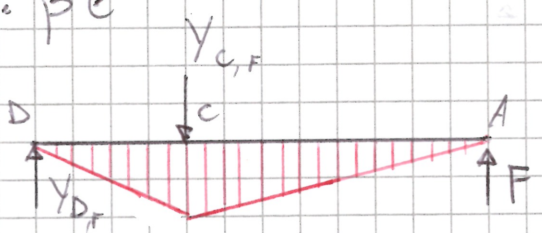
$X_{A_F} = 0$ perché guardo subito alla trave $\overline{DCA}$

$\hookrightarrow X_{B,F} = 0$

$\uparrow^{+})\quad Y_{A,F} = F$

$\overset{+}{\circlearrowleft} B)\quad C_{B,F} = F \cdot \beta l$

$\overline{DCA}$



$\uparrow^{+})\quad F + Y_{D,F} = Y_{C,F} \;\rightarrow\; Y_{C,F} = F\left(\frac{\alpha+1}{\alpha}\right)$

$\overset{+}{\rightarrow})\quad X_{A,F} = 0$

$\overset{+}{\circlearrowleft} C)\quad Y_{D,F} = \frac{F}{\alpha}$

$$M_{f_{F,AB}} = -F \cdot \tau$$

$$M_{f_{F,AC}} = +F \cdot s$$

$$M_{f_{F,DC}} = \frac{F}{\alpha} \cdot t$$

Applico il teorema di Castigliano per determinare F.

$$\delta_B = 0 \;\to\; \frac{\partial U}{\partial F} = 0$$

$$\Rightarrow U = ?$$

$$U = \int_0^{\beta l} \frac{1}{2EJ}\left(0 + (-F\cdot z)\right)^2 dz + \int_0^{l} \frac{\left(-\frac{1}{2} q s^2 + Fs\right)^2}{2EJ} ds + \int_0^{\alpha l} \frac{1}{2EJ}\left(-\frac{ql}{2\alpha}t + \frac{F}{\alpha}\cdot t\right)^2 dt$$

$$= \frac{1}{2EJ}\left(\int_0^{\beta l} F^2 z^2 dz + \int_0^{l}\left(\frac{1}{4}q^2 s^4 + F^2 s^2 - \cancel{2}\frac{1}{\cancel{2}} q F s^3\right) ds + \int_0^{\alpha l}\left(\frac{q^2 l^2}{4\alpha^2}t^2 + \frac{F^2}{\alpha^2}t^2 - 2\frac{ql}{2\alpha}t\cdot\frac{F}{\alpha}t\right) dt\right) =$$

$$= \frac{1}{2EJ}\left(F^2\frac{\beta^3 l^3}{3} + \frac{1}{4}q^2\frac{l^5}{5} + F^2\frac{l^3}{3} - qF\frac{l^4}{4} + \frac{q^2 l^2}{4\alpha^2}\cdot\frac{\alpha^3 l^3}{3} + \frac{F^2}{\cancel{\alpha^2}}\frac{\cancel{\alpha^3} l^3}{3} - \frac{\cancel{2} ql}{\cancel{2\alpha}}\cdot\frac{F}{\cancel{\alpha}}\cdot\frac{\cancel{\alpha^3} l^3}{3}\right)$$

$$\frac{\partial U}{\partial F} = 0 \;\to\; \frac{2F\beta^3 \cancel{l^3}}{3} + 0 + 2\frac{F\cancel{l^3}}{3} - q\frac{\cancel{l^4}}{4} + 0 + \frac{2F\cdot\alpha \cancel{l^3}}{3} - \frac{ql\cdot\alpha\cancel{l^3}}{3} = 0$$

$$\frac{2}{3}\beta^3 F + \frac{2}{3}F - \frac{ql}{4} + \frac{2F\cdot\alpha}{3} - \frac{ql\alpha}{3} = 0$$

$$F\left(\frac{2}{3}\beta^3 + \frac{2}{3}\alpha + \frac{2}{3}\right) = ql\left(\frac{1}{4} + \frac{\alpha}{3}\right)$$

$$F = ql\left(\frac{\frac{1}{4} + \frac{\alpha}{3}}{\frac{2}{3}\beta^3 + \frac{2}{3}\alpha + \frac{2}{3}}\right)$$