0.1 Beam axis and cross section definition

A necessary condition for identifying a portion of deformable body as a beam – and hence applying the associated framework – is that its centroidal curve is at least loosely recognizable.

Once such centroidal line has been roughly defined, locally perpendicular planes may be derived whose intersection with the body itself defines the local beam cross section. Then, the G center of gravity position may be computed for each of the local cross sections, thus defining a second, refined centroidal line. A potentially iterative definition for the beam centroidal axis[1] is hence obtained.

A rather arbitrary orientation may then be chosen for the centroidal curve.

A *local* cross-sectional reference system may be defined by aligning the normal z axis with the oriented centroidal curve, and by employing as the first in-section axis, namely x, the projection onto the cross-section plane of a given global $\underline{\mathrm{v}}$ vector, that is assumed to be not parallel to the beam axis.

The second in-section axis y is then derived, in order to obtain a local $Gxyz$ right-handed coordinate system, whose unit vectors are $\hat{\imath}, \hat{\jmath}, \hat{k}$.

Such construction of the local reference system for the beam branch is consistent with most the Finite Element (FE) codes.

If a thin walled profile is considered in place of a solid cross section member – i.e., the section wall midplane is recognizable too (see paragraph XXX below), then a curvilinear coordinate s may be defined that spans the in-cross-section wall midplane. Such in-cross-section wall midplane consists in a possibly multi-branched curve, which is parametrically defined by a pair of $x(s), y(s)$ functions, with s spanning the conventional $[0, l]$ interval.

In the case material is homogeneous along the wall thickness, the local thickness value $t(s)$ is some relevance, along with a local through-wall-thickness coordinate $r \in [-t(s)/2, +t(s)/2]$.

Such s, r, in-section coordinates based on the profile wall may be employed in place of their cartesian x, y counterparts, if favourable.

[1]here, centroidal curve, centroidal line, centroidal axis, or simply beam axis are treated as synonyms.

0.2 Joints and angular points

Beam axis may be discontinuous at sudden body geometry changes; a rigid body connection is ideally assumed to restrict the relative motion of the proximal segments.

Such rigid joint modeling may be extended to more complex n-way joints; if the joint finite stiffness is to be taken into account, it has to be described through the entries of a rank $6(n-1)$ symmetric square matrix [2].

At joints and at the beam axis angular points the cylindrical bodies obtained by sweeping the cross sections along the centroidal curve branches do usually overlap, and in general they only loosely mimic the actual deformable body geometry.

The results obtained through the local application of the elementary beam theory are of a problematic nature; they may at most be employed to scale the triaxial local stress/strain fields[3] that are evaluated resorting to more complex modelings.

0.3 Cross-sectional resultants for the spatial beam

At any point along the axis the beam may be notionally split, thus obtaining two facing cross sections, whose interaction is limited to three components of interfacial stresses, namely the axial normal stress σ_{zz} and the two shear components τ_{yz}, τ_{zx}.

Three force resultant components may be defined by integration along the cross section area, namely the normal force, the y- and the

[2] i.e., joint stiffness is unfortunately not a scalar value.

[3] The peak stress values obtained through the elementary beam theory may be profitably employed as *nominal stresses* within the stress concentration effect framework.

x- oriented shear forces, respectively defined as

$$N = \int_A \sigma_{zz} dA$$
$$Q_y = \int_A \tau_{yz} dA$$
$$Q_x = \int_A \tau_{zx} dA$$

Three moment resultant components may be similarly defined, namely the x- and y- oriented bending moments, and the torsional moment. However, if the centroid is the preferred fulcrum for evaluating the bending moments, the below discussed C shear center is employed for evaluating the torsional moment; the two points might coincide, e.g. if the cross section is twice symmetric, but they are distinct in general. We hence define

$$M_x \equiv M_{(G,x)} = \int_A \sigma_{zz} y dA$$
$$M_y \equiv M_{(G,y)} = -\int_A \sigma_{zz} x dA$$
$$M_t \equiv M_{(C,z)} = \int_A \left[\tau_{yz}(x - x_C) - \tau_{zx}(y - y_C)\right] dA$$

The applied vector associated to the normal force component $(G, N\hat{k})$ is located at the section center of gravity , whereas the shear force $(C, Q_x\hat{\imath} + Q_y\hat{\jmath})$ is supposed to act at the shear center; such convention decouples the energy contribution of force and moment components for the straight beam.

Cross section resultants may be obtained, based on equilibrium for a statically determinate structure. The ordinary procedure consists in

- notionally splitting the structure at the cross section whose resultants are under scrutiny;
- isolating a portion of the structure that ends at the cut, whose locally applied loads are all known; the structure has to be preliminarily solved for the all the constraint reactions that act on the isolated portion;

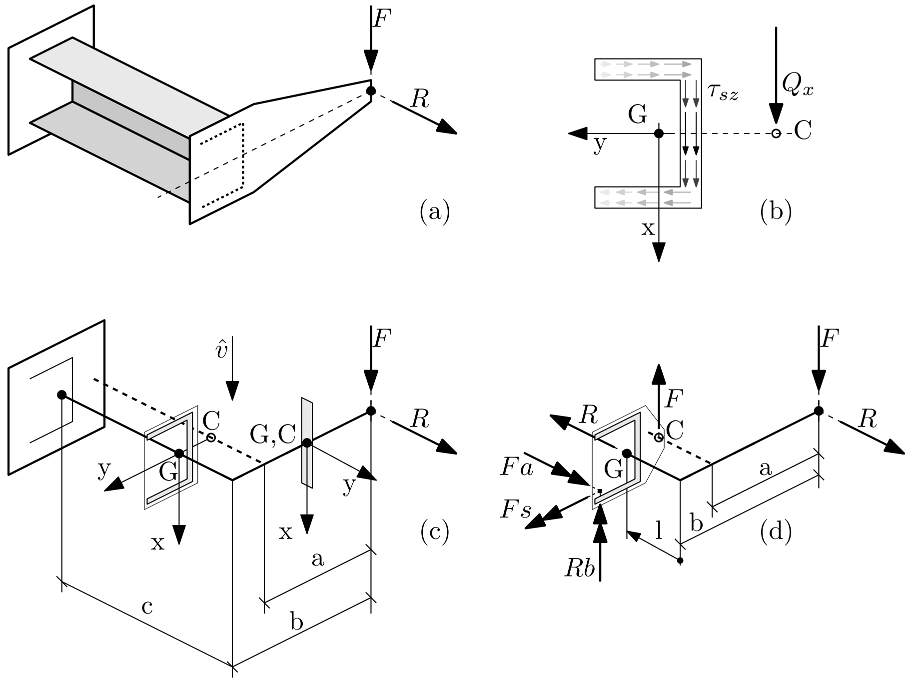

Figure 1: A beam structure.

- setting the equilibrium equations for the isolated substructure, according to which the cross-sectional resultants are in equilibrium with whole loading.

0.4 A worked example

See Figure 1. TODO.